Int. J. Heat Mass Transfer. Vol. 32, No. 6, pp. 1053–1061, 1989
Printed in Great Britain

0017-9310/89 \$3.00+0.00

Pressure and thermal effects in liquid metal due to collapse of external magnetic field

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(*Received* 4 *July* 1988)

Abstract—The equations under the MHD approximation show that a non-uniform magnetic field induces a pressure gradient and a current in a conducting liquid. The pressure difference and the change of temperature created due to the collapse of an external magnetic field are assessed. Considerable values of these effects lead to the conclusion that the effects in question can be of importance for future nuclear fusion power plants, where a liquid metal is a possible coolant.

1. INTRODUCTION

As IT IS well known, no thermonuclear power plant has yet been built. Some research work has been carried out but the energy balances of systems in action still remain negative.

Liquid metal is considered to be a possible coolant in such a power plant because of its suitable thermophysical properties. Inasmuch as the hot plasma has to be separated from material walls by means of magnetic traps, the entire system is to be exposed to the action of a strong magnetic field. Variations of this field can result for instance from an accident. This is why the effects of collapse of the external magnetic field in the liquid metal have been considered worthy of examination. Obviously, since the shape and the parameters of the cooling system in the future thermonuclear power plant are as yet unknown, this examination deals with physical phenomena involved rather than with their effects within a real construction.

Under the conditions of interest to the present analysis the MHD approximation is valid. The basic equations for liquid metals under this approximation are shown in Appendix A after ref. [1]. In Appendix A the dimensionless quantities and the similarity numbers, applied to the presented analysis, are defined as well.

The equation of motion, equation (A7), shows that a non-uniform magnetic field induces a pressure gradient in the liquid metal. According to Maxwell's equation (A2) a current is induced as well, which leads to Joule's heat generation and to an increase of the metal temperature. The aim of this paper is to assess the magnitude of these effects in the case of collapse of the external magnetic field.

The following problem will be considered. A circular cross-sectional tube filled with liquid metal at rest is surrounded by an ideal insulator, e.g. free space, and finds itself in a magnetic field which, being initially uniform and equal to $\mathbf{B}_0$, drops far from the tube according to

$$\mathbf{B}_{\mathrm{M}} = \mathbf{B}_0 \exp\left(-\frac{t}{t_0}\right); \quad \hat{\mathbf{B}}_{\mathrm{M}} = \exp\left(-\frac{\tau}{\tau_0}\right) \quad (1)$$

where t_0 is the characteristic time of the electromagnet. The tube is long and by virtue of this fact all the variables of interest are independent of the z-coordinate, i.e. they are constant along the tube axis. Such a system seems to be a convenient choice in so far as both an analytical solution can be found and the results may be of use for the future applications.

Two limiting cases will be considered:

(a) the external magnetic field parallel to the tube axis (Appendix B);

(b) the external magnetic field perpendicular to the tube axis (Appendix C).

Any other case is completely specified by the two limiting cases given above. The pressure variations, the heat generation rate and the change of temperature will be assessed on the grounds of these magnetic field distributions.

2. THE PRESSURE VARIATIONS

2.1. *The magnetic field parallel to the tube axis*

The problem is axisymmetrical and, according to equation (B1), the Lorentz body force reduces to the magnetic pressure gradient. Consequently, for liquid at rest the equation of motion, equation (A7), reduces to the magnetic Bernoulli equation

$$\nabla\left(p+\frac{B^2}{2\mu_0}\right) = 0. \quad (2)$$

The magnetic field distribution is evaluated in Appendix B. Integrating equation (2) yields

NOMENCLATURE

- $\mathbf{A}$ vector potential
- $\mathbf{A}_l$ vector potential for $\lambda = 0$
- a thermal diffusivity
- $\mathbf{B}$ magnetic induction
- $\mathbf{B}_0$ initial magnetic induction
- $\mathbf{B}_l$ magnetic induction for $\lambda = 0$
- $\mathbf{B}_L$ non-homogeneous part of magnetic induction, cf. equation (B9)
- $\mathbf{B}_M$ magnetic induction far from the tube
- b dimensionless parameter, cf. equation (C13)
- C_0, G, g, F, f auxiliary functions
- c_p specific heat at constant pressure
- D_n, K_n, P_n coefficients of orthogonal expansions
- $\mathbf{E}$ electric field
- Fo Fourier number
- H inductance, cf. equation (C8)
- h heat transfer coefficient
- Ha Hartman number
- $\mathbf{I}$ resultant current
- $\mathbf{i}$ current density
- J_0, J_1 Bessel functions
- k thermal conductivity
- L characteristic length
- Nu Nusselt number
- p pressure
- Δp pressure difference
- Δp_{MAX} maximum of Δp for given τ_0
- Pe Peclet number
- Pr Prandtl number
- Pr_{m} magnetic Prandtl number
- Pr'_{m} modified magnetic Prandtl number
- Q heat generation per unit length
- $\dot{Q}$ heat generation rate per unit length
- $\mathbf{q}$ heat flux
- q_v heat generation rate per unit volume
- R resistance, cf. equation (C8)
- Re Reynolds number
- Re_{m} magnetic Reynolds number
- r, θ, z polar coordinates
- r_0 tube radius
- S characteristic area, cf. equation (C9)
- T temperature
- T_L liquid temperature
- T_{MAX} maximum temperature over the tube cross-section
- T_W wall temperature
- t time
- t_0 time constant, cf. equation (1)
- t_1 time constant, cf. equation (C10)
- U characteristic velocity
- $\mathbf{u}$ velocity
- x, y Cartesian coordinates.

Greek symbols

- α_n, β_n roots of Bessel functions, cf. equations (B6) and (C14)
- ε electromotive force
- η magnetic diffusivity
- λ auxiliary constant, cf. equation (C3)
- μ_0 magnetic permeability of free space
- ν kinematic viscosity
- ρ density
- σ conductivity
- τ dimensionless time
- τ_0, τ_1 dimensionless time constants corresponding to t_0 and t_1, respectively
- ϕ magnetic flux
- φ, ψ auxiliary functions.

The same letter may denote a vector (bold) or its modulus as a scalar (italic). Dimensionless quantities are referred to with a ^.

$$\Delta\hat{p}(\hat{r},\tau) = \hat{p}(\hat{r},\tau) - \hat{p}(\hat{r}=0,\tau) = \hat{B}^2(\hat{r}=0,\tau) - \hat{B}^2(\hat{r},\tau). \quad (3)$$

The pressure distributions for various values of τ are shown in Fig. 2 for $\tau_0 = 0.1$. At every instant the greatest $\Delta\hat{p}$ occurs at the wall. The pressure at the wall reaches a maximum in a relatively short time and the value of this maximum $\Delta\hat{p}_{\mathrm{MAX}}$ strongly depends on τ_0, which is shown in Fig. 3.

The dimensionless value $\Delta\hat{p} = 1.0$ is equivalent to $3.98 \times 10^5 B_0^2$ in SI units, which for $B_0 = 5$ T yields Δp virtually equal to 10 MPa (cf. equation (A14)).

The pressure difference Δp_{MAX} is relatively high for small τ_0 only. It is equivalent to a short time t_0 as well as to large values of conductivity or of the tube radius. For liquid sodium for instance, taking $r_0 = 0.1$ m. $\tau = 1.0$ refers to $t \simeq 0.1$ s [2].

2.2. *The magnetic field perpendicular to the tube axis*

Equation (2) is now not as rigorous as in the preceding case. Nevertheless, it can be applied, at least for the approximate analysis. The brief discussion of this problem is given at the end of Appendix C. In Appendix C the suitable magnetic field is evaluated as well.

This field, and consequently the pressure distribution, depend not only on τ_0, but on parameters τ_1 and b as well. These parameters refer to the unknown geometry of the cooling system, part of which is the considered tube—cf. equations (C9)–(C11) and

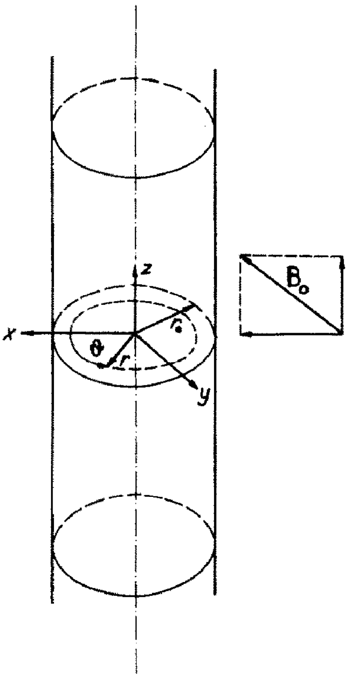

FIG. 1. The considered system.

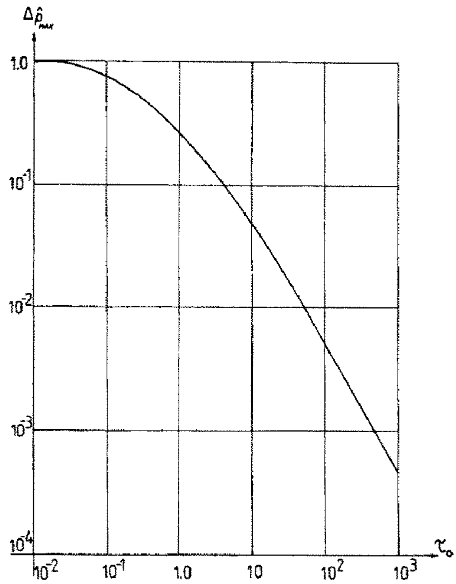

FIG. 3. The dimensionless maximum pressure at the wall $\Delta\hat{p}_{MAX}$ vs τ_0 (parallel external field). $\Delta\hat{p} = 1.0$ is equivalent to $\Delta\hat{p} = 10$ MPa for $B_0 = 5$ T.

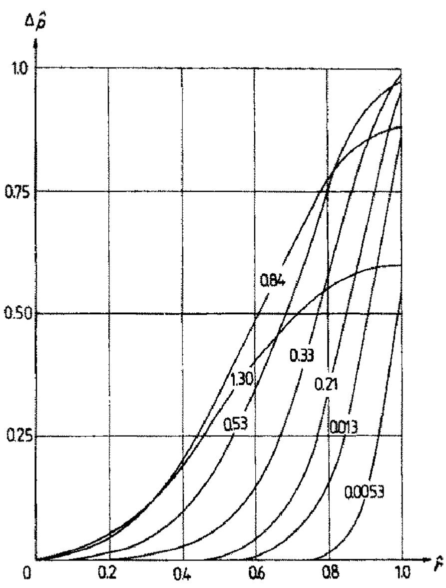

FIG. 2. The dimensionless pressure distribution for various values of the dimensionless time τ when $\tau_0 = 0.013$ (parallel external field).

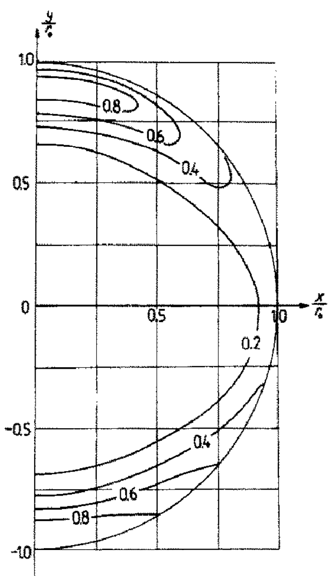

FIG. 4. The isobars $\Delta\hat{p}$ = const. for $\tau = 0.02$ (perpendicular external field).

(C13)—and their values can be estimated only by intuition. The results presented in Figs. 4 and 5 were obtained for $b = 1.0$, $\tau_0 = 0.01$, $\tau_1 = 3.0$.

The isobaric lines are shown in Fig. 4. The pressure distribution for $x = 0$ is shown in Fig. 5 for various values of τ. It can be seen that the order of magnitude is the same as that for the parallel external field. This remark holds in a wide range of values of τ_1 and b. A remarkable effect is noted. At successive stages the pressure becomes greater at the axis than at the wall. This 'pinch' due to resultant current flow is not of great value but may be of significance, e.g. in a case of boiling. The influence of the resultant current increases when $\tau_1 \to \tau_0$.

The case of an arbitrary orientated external magnetic field can be examined as the proper superposition of the cases given above.

3. THE THERMAL EFFECTS

The current density can be evaluated from Maxwell's equation (A2). The heat generation rate per unit volume due to this current is described by

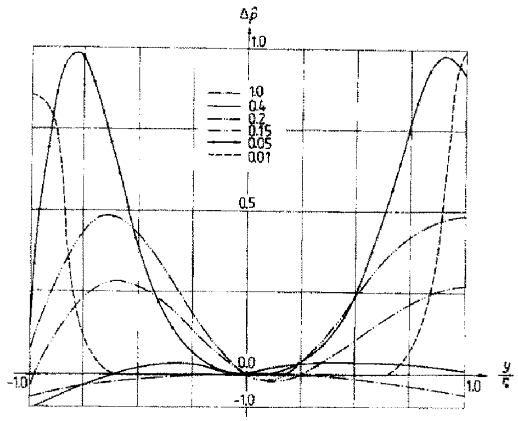

FIG. 5. The dimensionless pressure distribution along the diameter perpendicular to the initial magnetic field for various values of the dimensionless time τ (perpendicular external field).

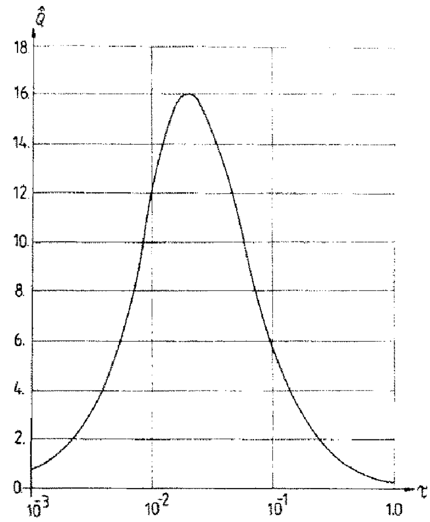

FIG. 6. The dimensionless heat generation rate per 1 m of the tube length $\hat{Q}$ vs the dimensionless time τ.

$$q_v = \frac{i^2}{\sigma}. \tag{4}$$

Assuming thermophysical properties to be constant and considering equations (A17) and (A18) one can write the dimensionless heat conduction equation as follows:

$$\frac{\partial \hat{T}}{\partial \tau} = \frac{\mu_0 \sigma k}{c_p \rho} \hat{\nabla}^2 \hat{T} + \hat{i}^2. \tag{5}$$

A certain interpretation of the coefficient in equation (5) may be proposed. Substituting the kinematic viscosity ν for the magnetic diffusivity $\eta = (\mu_0 \sigma)^{-1}$ in Reynolds number $Re = Ur_0/\nu$ one obtains the magnetic Reynolds number $Re_m = Ur_0\mu_0\sigma$. The definition of the modified magnetic Prandtl number Pr'_m may be introduced in the same way

$$Pr = \frac{\nu}{a}$$

$$Pr'_m = \frac{\eta}{a} = \frac{c_p \rho}{\mu_0 \sigma k}. \tag{6}$$

The heat conduction equation (5) is now

$$\frac{\partial \hat{T}}{\partial \tau} = \frac{1}{Pr'_m} \hat{\nabla}^2 \hat{T} + \hat{i}^2. \tag{7}$$

One should note that there also exists a classical definition of the magnetic Prandtl number $Pr_m = \nu/\eta = Re_m/Re$ (cf. ref. [1]). Each one seems to be appropriate in different problems. The number Pr_m is unrelated to any heat transfer whereas the number Pr'_m introduced above and the ordinary Pr are in a sense symmetrical. They both refer to heat transfer. Moreover, the following relation occurs:

$$Pe = Re\,Pr = Re_m\,Pr'_m.$$

Taking liquid sodium as an example one obtains at $T = 400°C$, $Pr'_m = 2200$ [2].

Equation (7) has been solved numerically in ref. [3]. The computer code COMES, by J. Banaszek, based on the CVFE method has been applied (for details of the method see ref. [4]). The initial temperature was assumed to be constant

$$\hat{T}(\hat{r}, \theta, \tau = 0) = 0.$$

The adiabatic wall was taken as the boundary condition

$$\left.\frac{\partial \hat{T}}{\partial \hat{r}}\right|_{\hat{r}=1} = 0. \tag{8}$$

There are no data, on the grounds of which a real boundary condition referring to a thermonuclear reactor and its cooling system can be submitted. Assumption (8) results in the greatest possible excess of temperature due to effects in question. The symmetry of the current density leads to the second boundary condition

$$\left.\frac{\partial \hat{T}}{\partial \hat{x}}\right|_{\hat{x}=0} = 0.$$

The results presented below were obtained in the case of an external magnetic field perpendicular to the tube axis for $b = 1.0$, $\tau_0 = 0.01$, $\tau_1 = 3.0$ and $Pr'_m = 2200$. The exemplifying dimensional values refer to liquid sodium at $T = 400°C$ and to $r_0 = 10$ cm. The denominator in relation (A16) is then equal to 0.07. The total heat generation rate per metre of the tube length $\hat{Q}$ vs time is shown in Fig. 6 (cf. equation (A19))

$$\dot{Q} = \int_V q_v \, dV. \tag{9}$$

The maximum value of the heat generation rate per unit volume is of the order of 10^9 W m^{-3} for $B_0 = 1$ T. The maximum value of $\hat{Q}$ is equivalent to 3.76 MW m^{-1} for $B_0 = 1$ T and to 94 MW m^{-1} for $B_0 = 5$ T. The heat rate of such a great value is generated, however, for a relatively short period. Consequently the total heat generation is of moderate value

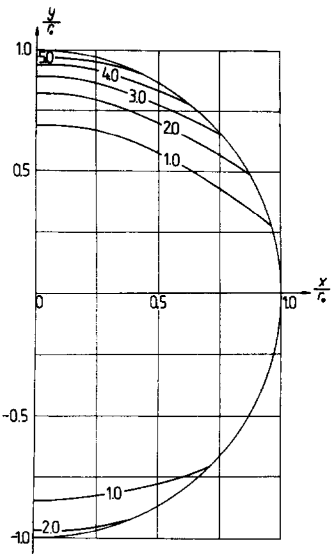

FIG. 7. The isotherms $\hat{T}$ = const. for $\tau = 0.1$.

$$Q = \int_0^t \dot{Q}(t')\,dt'. \tag{10}$$

For $\tau = 0.02$, i.e. for maximum $\hat{\dot{Q}}$, $\hat{\dot{Q}} = 0.466$. It is equivalent to 3.72 kJ m^{-1} for $B_0 = 1$ T and to 93 kJ m^{-1} for $B_0 = 5$ T. For $\tau = 1.0$, $\hat{Q} = 3.666$, which is tantamount to 29.2 and 730 kJ m^{-1}, respectively (cf. equation (A20)).

A temperature distribution is shown in Fig. 7. At every instant the highest temperature occurs at the wall for $x = 0$. This maximum $\hat{T}_{\text{MAX}}$ vs time is shown in Fig. 8. The highest value $\hat{T}_{\text{MAX}} = 6.78$ (for τ approximately equal to 0.6) is equivalent to the increase of temperature $\Delta T = 5$ K for $B_0 = 1$ T and $\Delta T = 125$ K for $B_0 = 5$ T.

The excess of temperature of such a value, together with the foregoing change of pressure, evidently can modify the heat transfer conditions. Hence, the entire analysis of heat transfer in a liquid metal in a strong magnetic field has to include the effects in question.

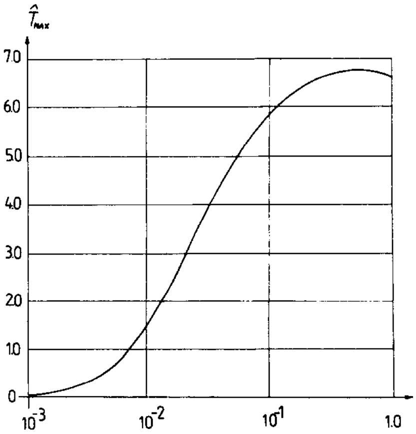

FIG. 8. The dimensionless maximum temperature $\hat{T}_{\text{MAX}}$ vs the dimensionless time τ.

On the other hand any complete description of heat transfer in a liquid metal coolant requires convection being included. To solve the MHD flow together with the heat transfer in the varying external magnetic field is, however, no easy task and both numerical and experimental modelling seem to be necessary. The dimensional analysis can be useful as well.

The well-known relation, describing the convectional heat flux density q, is of shape

$$q = h(T_W - T_L).$$

The heat transfer coefficient h can be evaluated by means of relations, mainly empirical, between proper similarity numbers. The change of the heat flux due to considered effects may possibly be included in the same way. To this purpose the adequate set of similarity numbers has to be formed first of all.

Inspection of the Appendices leads to the following set of relevant quantities: initial magnetic induction B_0, magnetic diffusivity $\eta = (\mu\sigma)^{-1}$, electrical conductivity σ, time constant t_0, t_1 (or both of them if necessary). Considering also the classical case of convectional heat transfer one can assume

$$h = h(k, r_0, U, \rho, \rho v, c_p, \eta, t_0, \sigma, B_0).$$

There are 11 dimensional quantities and 5 fundamental units (kg, m, s, K, A). Consequently, one can form the set of six dimensionless numbers

$$Nu = Nu\left(\frac{v\rho c_p}{k}, \frac{Ur_0}{\eta}, \frac{\eta\rho c_p}{k}, \frac{t_0}{\mu_0\sigma r_0^2}, \frac{c_p\sigma r_0^2 B_0^2}{k}\right). \tag{11}$$

Relation (11) can be expressed by means of the similarity numbers as follows (cf. equations (A21)–(A29)):

$$Nu = Nu(Re, Pr, Pr'_m, Fo, Ha). \tag{12}$$

The Fourier number Fo may refer both to the time constant t_0 connected with the external magnetic field and to t_1, the time constant of the system taken as an electrical circuit.

Equation (12) seems to be of use for any experimental modelling of the effects in question.

4. CONCLUSIONS

Considering a liquid metal as a possible coolant in a nuclear fusion power plant one should take into account the analysed effects. The order of magnitude of the heat generation and the pressure variations is such that both the heat transfer can be influenced and the considerable force interaction between the liquid and the wall can occur. It depends first of all on the initial magnetic field B_0 and on the time constants τ_0 and τ_1. The geometry of the system is of consequence as well.

To obtain a complete description of the problem, the flow of a liquid metal and the real geometry of a cooling system have to be included, which requires both numerical and experimental research.

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APPENDIX A. EQUATIONS UNDER THE MHD APPROXIMATION AND DIMENSIONLESS QUANTITIES

A1. *The MHD approximation*

The liquid metal flow which undergoes an interaction with a magnetic field is of such a nature that it allows the so-called MHD (MagnetoHydroDynamics) approximation [1]. The following basic assumptions are made in addition to the non-relativistic conditions.

(1) The induced magnetic field is much smaller than the externally applied magnetic field. For liquid metal the magnetic permeability is usually assumed to be equal to that of free space.

(2) Phenomena involving high frequency are not taken into account so that the displacement current is neglected compared to the conduction one.

In liquid conductors the space charge may be neglected. The basic equations for the liquid metal under the MHD approximation are as follows:

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{A1}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{i} \tag{A2}$$

$$\nabla\cdot\mathbf{i} = 0 \tag{A3}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{A4}$$

$$\mathbf{i} = \sigma(\mathbf{E}+\mathbf{u}\times\mathbf{B}) \tag{A5}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{A6}$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p+\nu\nabla^2\mathbf{u}-\frac{1}{\rho}\left[\nabla\left(\frac{B^2}{2\mu_0}\right)-\frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\mathbf{B}\right]. \tag{A7}$$

Equations (A1)–(A4) are Maxwell's equations, (A5) is Ohm's law, (A6) is the continuity equation for incompressible fluid and (A7) is the equation of motion with the Lorentz body force. D/Dt denotes the substantial time derivative.

Assuming both the magnetic permeability and the electrical conductivity to be constant the following relation can be derived from equations (A1), (A2), (A4) and (A5):

$$\frac{\partial\mathbf{B}}{\partial t} = \eta\nabla^2\mathbf{B}+\nabla\times(\mathbf{u}\times\mathbf{B}) \tag{A8}$$

where $\eta = (\mu_0\sigma)^{-1}$ is called the magnetic diffusivity. This relation, along with equation (A3), contains all the information about the magnetic field and, together with equations (A6) and (A7), completely specifies the problem. The magnetic Reynolds number $Re_{\mathrm{m}} = UL\mu_0\sigma$ is a measure of the ratio of magnetic convection to magnetic diffusion [1]. In the case of $Re_{\mathrm{m}} \ll 1$ diffusion is the dominant means of transport and the term $\nabla\times(\mathbf{u}\times\mathbf{B})$, which represents in equation (A8) a pure convection, is to be neglected. It leads to the magnetic diffusion equation

$$\frac{\partial\mathbf{B}}{\partial t} = \eta\nabla^2\mathbf{B}. \tag{A9}$$

A2. *The dimensionless quantities*

The dimensionless quantities, applied to the presented analysis, are defined as follows:

$$\hat{A} = \frac{A}{B_0 r_0} \tag{A10}$$

$$\hat{B} = \frac{B}{B_0} \tag{A11}$$

$$\hat{i} = i\frac{\mu_0 r_0}{B_0} \tag{A12}$$

$$\hat{I} = I\frac{\mu_0 L}{nB_0 S}. \tag{A13}$$

The definitions of S and L are given by equations (C9) and (C11)

$$\hat{p} = p\frac{2\mu_0}{B_0^2} \tag{A14}$$

$$\hat{r} = \frac{r}{r_0} \tag{A15}$$

$$\tau = \frac{t}{\mu_0\sigma r_0^2} \tag{A16}$$

$$\hat{T} = T\frac{\mu_0 c_p\rho}{B_0^2} \tag{A17}$$

$$\hat{q}_v = \hat{i}^2 = q_v\frac{\mu_0^2\sigma r_0^2}{B_0^2} \tag{A18}$$

$$\hat{\dot{Q}} = \dot{Q}\frac{\mu_0^2\sigma}{2B_0^2} \tag{A19}$$

$$\hat{Q} = Q\frac{\mu_0}{r_0^2 B_0^2} \tag{A20}$$

$$Fo = \frac{at_0}{r_0^2} \tag{A21}$$

$$Ha = \sqrt{\left(\frac{\sigma}{\rho\nu}\right)}B_0 r_0 \tag{A22}$$

$$Nu = \frac{hr_0}{k} \tag{A23}$$

$$Pe = \frac{Ur_0}{a} \tag{A24}$$

$$Pr = \frac{\nu}{a} \tag{A25}$$

$$Pr_{\mathrm{m}} = \frac{\nu}{\eta} \tag{A26}$$

$$Pr'_{\mathrm{m}} = \frac{\eta}{a} \tag{A27}$$

$$Re = \frac{Ur_0}{\nu} \tag{A28}$$

$$Re_{\mathrm{m}} = \frac{Ur_0}{\eta}. \tag{A29}$$

APPENDIX B. THE EXTERNAL MAGNETIC FIELD PARALLEL TO THE TUBE AXIS

The problem is axisymmetrical (Fig. 1). The radial and circumferential components of the magnetic induction vector are equal to zero

$$\mathbf{B} = \mathbf{1}_z B(r,t). \tag{B1}$$

Thus, for liquid being initially at rest, the magnetic transport equation (A8) reduces to

$$\frac{\partial \hat{B}}{\partial \tau} = \frac{\partial^2 \hat{B}}{\partial \hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial \hat{B}}{\partial \hat{r}}. \tag{B2}$$

Maxwell's equations (A1) and (A2) together with Ohm's law (A5) give

$$\frac{1}{\hat{r}}\frac{\partial}{\partial \hat{r}}(\hat{r}\hat{i}_\theta) = -\frac{\partial \hat{B}}{\partial \tau}$$

$$-\frac{\partial \hat{B}}{\partial \hat{r}} = \hat{i}_\theta.$$

From the relations given above it appears that the considered magnetic field induces in the tube a current with the circumferential component only. The situation is therefore similar to that in an infinite solenoid. The magnetic field induced by such an axisymmetrical current remains inside the tube and does not disturb the external magnetic field. This leads to the following boundary condition:

$$\hat{B}(\hat{r}=1,\tau) = \hat{B}_M = \exp\left(-\frac{\tau}{\tau_0}\right). \tag{B3}$$

The second boundary condition results from the continuity of the field $\mathbf{B}$ at the tube axis. The initial condition is

$$\hat{B}(\hat{r},\tau=0) = 1. \tag{B4}$$

The solution of equation (B2) under the conditions given above is to be found in the shape of a sum

$$\hat{B}(\hat{r},\tau) = g(\hat{r},\tau) + \varphi(\hat{r},\tau) \tag{B5}$$

where the auxiliary functions g and φ fulfil equation (B2) but with different initial and boundary conditions, namely

$$g(\hat{r},\tau=0) = 1, \quad \varphi(\hat{r},\tau=0) = 0$$

$$g(\hat{r}=1,\tau) = 0, \quad \varphi(\hat{r}=1,\tau) = \exp\left(-\frac{\tau}{\tau_0}\right).$$

Considering that (cf. ref. [5])

$$\sum_{n=1}^{\infty} \frac{J_0(\alpha_n \hat{r})}{\alpha_n J_1(\alpha_n)} = \frac{1}{2}$$

$$J_0(\alpha_n) = 0 \tag{B6}$$

one obtains

$$g(\hat{r},\tau) = 2\sum_{n=1}^{\infty} \frac{J_0(\alpha_n \hat{r})}{\alpha_n J_1(\alpha_n)} \exp(-\alpha_n^2 \tau). \tag{B7}$$

The function φ can be found by means of Duhamel's theorem (see ref. [6]). Let $f(\hat{r},\tau)$ fulfil equation (B2) under the following conditions:

$$f(\hat{r},\tau=0) = 0, \quad f(\hat{r}=1,\tau) = 1.$$

Then

$$\varphi(\hat{r},\tau) = \int_0^\tau \exp\left(-\frac{\tau'}{\tau_0}\right)\frac{\partial}{\partial \tau} f(\hat{r},\tau-\tau')\,d\tau'.$$

The function f is given by

$$f = 1 - g$$

and upon integrating one obtains

$$\varphi = 2\sum_{n=1}^{\infty} \frac{J_0(\alpha_n \hat{r})}{J_1(\alpha_n)} \frac{\alpha_n \tau_0}{(\alpha_n^2 \tau_0 - 1)}\left[\exp\left(-\frac{\tau}{\tau_0}\right) - \exp(-\alpha_n^2 \tau)\right]. \tag{B8}$$

Relations (B5), (B7) and (B8) specify completely the magnetic field in question. Simple transformations give

$$\hat{B}(\hat{r},\tau) = \hat{B}_M(\tau) + \hat{B}_L(\hat{r},\tau) \tag{B9}$$

where $\hat{B}_M$ is the homogeneous external field described by relation (1) while the non-homogeneous part $\hat{B}_L$ is

$$\hat{B}_L = 2\sum_{n=1}^{\infty} J_0(\alpha_n \hat{r}) K_n \left[\exp\left(-\frac{\tau}{\tau_0}\right) - \exp(-\alpha_n^2 \tau)\right]$$

$$K_n = [\alpha_n J_1(\alpha_n)(\alpha_n^2 \tau_0 - 1)]^{-1}.$$

APPENDIX C. THE EXTERNAL MAGNETIC FIELD PERPENDICULAR TO THE TUBE AXIS

The case is more complicated than that analysed in Appendix B. The problem is two-dimensional and the Lorentz body force has a non-potential part. Consequently, even for liquid being initially at rest a velocity field can be induced. From equations (A7) and (A8) it appears that in general the velocity and the magnetic induction are not to be calculated separately. One can apply, however, equation (A9) instead of equation (A8) in the case $Re_m \ll 1$. As the induced velocity is expected to be of moderate value, this restriction has been assumed in the present analysis.

For liquid sodium as an instance, assuming a characteristic velocity U of the order of a few centimetres per second and a radius r_0 of the order of a few centimetres, one obtains $Re_m \simeq 10^{-2}$.

In the two-dimensional problem the induced magnetic field affects the external one. Both the tube and the surroundings have to be considered simultaneously. This leads to the following boundary condition at the tube wall:

$$\mathbf{B}(\hat{r}\to 1^+,\theta,\tau) = \mathbf{B}(\hat{r}\to 1^-,\theta,\tau).$$

The initial condition and the other boundary conditions are the same as in Appendix B.

The vector potential $\mathbf{A}$ may be introduced as follows (refs. [1, 7]):

$$\nabla\times\mathbf{A} = \mathbf{B}$$

$$\nabla\cdot\mathbf{A} = 0$$

$$\mathbf{A} = \mathbf{1}_z A$$

$$\hat{B}_r = \frac{1}{\hat{r}}\frac{\partial \hat{A}}{\partial \theta}$$

$$\hat{B}_\theta = -\frac{\partial \hat{A}}{\partial \hat{r}}.$$

For $\hat{r} \leqslant 1$ the magnetic diffusion equation (A9) yields

$$\frac{\partial \hat{\mathbf{A}}}{\partial \tau} = \hat{\nabla}^2 \hat{\mathbf{A}}. \tag{C1}$$

From Maxwell's equation (A2) there is for $\hat{r} \geqslant 1$

$$\hat{\nabla}^2 \hat{\mathbf{A}} = 0. \tag{C2}$$

The initial condition leads to

$$\hat{A}(\hat{r},\theta,\tau=0) = \hat{r}\sin\theta.$$

Taking into account equation (1) and the essential continuity of the fields in question at the wall, the set of boundary conditions may be established as follows:

$$\left.\frac{\partial \hat{A}}{\partial \theta}\right|_{\hat{r}\to\infty} = \hat{r}\exp\left(-\frac{\tau}{\tau_0}\right)\cos\theta$$

$$\left.\frac{\partial \hat{A}}{\partial \hat{r}}\right|_{\hat{r}\to\infty} = \exp\left(-\frac{\tau}{\tau_0}\right)\sin\theta$$

$$\hat{A}(\hat{r}\to 1^+,\theta,\tau) = \hat{A}(\hat{r}\to 1^-,\theta,\tau)$$

$$\left.\frac{\partial \hat{A}}{\partial \theta}\right|_{\hat{r}\to 1^+} = \left.\frac{\partial \hat{A}}{\partial \theta}\right|_{\hat{r}\to 1^-}$$

$$\left.\frac{\partial \hat{A}}{\partial \hat{r}}\right|_{\hat{r}\to 1^+} = \left.\frac{\partial \hat{A}}{\partial \hat{r}}\right|_{\hat{r}\to 1^-}.$$

The continuity of the magnetic field at the tube axis is also required. The separation of the variables in equation (C1) leads to

$$\hat{A} = G(\tau)F(\hat{r})\psi(\theta)$$

$$\hat{r}^2\frac{G'}{G} - \hat{r}^2\frac{1}{F}\left(\frac{1}{\hat{r}}F' + F''\right) = \frac{\psi''}{\psi} = \text{const.} = -\lambda^2. \quad \text{(C3)}$$

C1. *The case* $\lambda \neq 0$

Separating the variables in equation (C2) and applying Duhamel's theorem one obtains:

for $\hat{r} \leqslant 1$

$$\hat{A} = \left\{\hat{r}\exp\left(-\frac{\tau}{\tau_0}\right) + \sum_{n=1}^{\infty} D_n \frac{J_1(\alpha_n \hat{r})}{J_1(\alpha_n)}\left[\exp\left(-\frac{\tau}{\tau_0}\right) - \exp(-\alpha_n^2\tau)\right]\right\}\sin\theta \quad \text{(C4)}$$

$$D_n = 4[\alpha_n^2(\alpha_n^2\tau_0 - 1)]^{-1};$$

for $\hat{r} \geqslant 1$

$$\hat{A} = \left\{\hat{r}\exp\left(-\frac{\tau}{\tau_0}\right) + \frac{1}{\hat{r}}\sum_{n=1}^{\infty} D_n\left[\exp\left(-\frac{\tau}{\tau_0}\right) - \exp(-\alpha_n^2\tau)\right]\right\}\sin\theta. \quad \text{(C5)}$$

The above relations describe both the magnetic induction field and the induced current. For $\hat{r} \leqslant 1$ there is

$$\hat{\nabla}^2\hat{A} = -\hat{i}$$

$$\hat{i} = -\frac{\partial \hat{A}}{\partial \tau}.$$

It is evident that the symmetries of vectors **A** and **i** are alike. It leads to the conclusion that the resultant current is equal to zero for such a potential.

C2. *The case* $\lambda = 0$

Equation (C2) yields

$$\hat{A}_I = -C_0(\tau)\ln\hat{r} + D_0(\tau).$$

This axisymmetrical vector potential $\hat{A}_I$ fulfils the boundary condition far from the tube and results in the magnetic field having the circumferential component only

$$B_I = \frac{C_0}{\hat{r}}. \quad \text{(C6)}$$

Applying Stokes' theorem and integrating over the circle for $\hat{r} > 1$ one obtains

$$2\pi r B_I = \mu_0 I(\tau)$$

$$C_0 = \frac{\mu_0}{2\pi r_0} I(\tau). \quad \text{(C7)}$$

Thus relation (C6) describes the axisymmetrical field B_I induced due to resultant current I flowing along the tube. Such a current cannot flow in the infinitely long tube and is not to be specified in terms of the plane problem. Considering however the tube in question as an idealization of a part of a cooling system one has to take into account the fact that such a system—filled with conducting liquid—forms an electrical circuit. The variation of the magnetic flux results in a current flowing in the circuit.

Let a cooling system be an electrical circuit of inductance H and of resistance R. One should insist on the fact that the determination of these parameters requires knowledge of the geometry of the whole cooling system and therefore it can be made only very roughly.

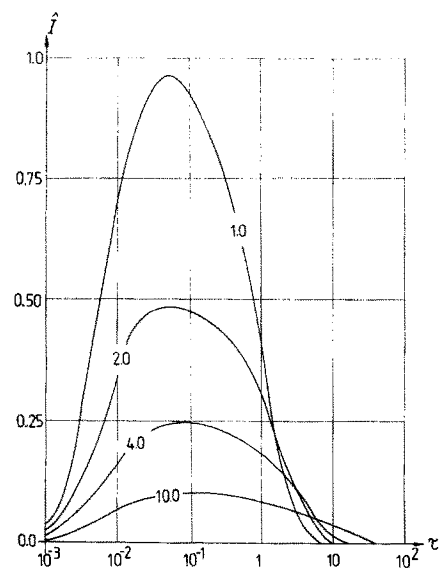

FIG. C1. The dimensionless resultant current $\hat{I}$ vs the dimensionless time τ for various values of τ_1 when $\tau_0 = 0.01$. $\hat{I} = 1.0$ is equivalent to the current of the order of 10^6 A for S/L of the order of 1 m when $B_0 = 1$ T.

The resultant current is then described by

$$\varepsilon - H\frac{dI}{dt} = RI; \quad \varepsilon = -\frac{d\phi}{dt} \quad \text{(C8)}$$

where ε stands for the electromotive force. The magnetic flux ϕ can be estimated as follows:

$$\phi = B_0 S\exp\left(-\frac{\tau}{\tau_0}\right) \quad \text{(C9)}$$

where S is a measure of an area surrounded by the circuit and perpendicular to the magnetic field. Introducing the circuit time constant

$$t_1 = \frac{H}{R}; \quad \tau_1 = \frac{t_1}{\mu_0\sigma r_0^2} \quad \text{(C10)}$$

and transforming the resistance R as follows:

$$R = \frac{L}{\pi r_0^2\sigma} \quad \text{(C11)}$$

where L stands for a measure of the circuit length one obtains under the initial condition $I(t = 0) = 0$ the following solution of equation (C8):

$$\hat{I}(\tau) = \frac{1}{\tau_1 - \tau_0}\left[\exp\left(-\frac{\tau}{\tau_1}\right) - \exp\left(-\frac{\tau}{\tau_0}\right)\right]. \quad \text{(C12)}$$

The values of S, L and τ_1 are unknown. According to some remarks in ref. [7] the time constant τ_1 is expected to be rather greater than τ_0.

The dimensionless current $\hat{I}$ vs time is shown in Fig. C1 for various values of τ_1. $\hat{I} = 1$ is equivalent to 2.5×10^6 A for $S/L = 1$ m and $B_0 = 1$ T (cf. equation (A13)).

The current evaluated above specifies the magnetic induction field for $r > 1$ according to equations (C6) and (C7). The field in the liquid is to be calculated from magnetic diffusion equation (A9). Relations (C6), (C7) and (C12) result in the boundary condition at the tube wall:

$$\hat{B}_I(\hat{r}=1,\tau) = \frac{b}{\tau_1-\tau_0}\left[\exp\left(-\frac{\tau}{\tau_1}\right)-\exp\left(-\frac{\tau}{\tau_0}\right)\right]$$

$$b = \frac{S}{2Lr_0}. \tag{C13}$$

The initial condition is evident

$$\hat{B}_I(\hat{r},\tau=0) = 0.$$

Separating the variables and applying the proper orthogonal expansions one obtains

$$\hat{B}_I(\hat{r},\tau) = \frac{b}{\tau_1-\tau_0}\left[\frac{J_1(\hat{r}\tau_1^{-1/2})}{J_1(\tau_1^{-1/2})}\exp\left(-\frac{\tau}{\tau_1}\right)\right.$$
$$\left.-\frac{J_1(\hat{r}\tau_0^{-1/2})}{J_1(\tau_0^{-1/2})}\exp\left(-\frac{\tau}{\tau_0}\right)+\sum_{n=1}^{\infty}P_nJ_1(\beta_n\hat{r})\exp(-\beta_n^2\tau)\right]$$

$$P_n = \frac{2\beta_n}{J_0(\beta_n)}\left(\frac{\tau_0}{1-\tau_0\beta_n^2}-\frac{\tau_1}{1-\tau_1\beta_n^2}\right)$$

$$J_1(\beta_n) = 0. \tag{C14}$$

The magnetic induction field in liquid metal for the resultant current $I \neq 0$ is given by the superposition of the circumferential field B_I and of that resulting from equation (C4).

To be precise one should note that certain terms in the relations given above as well as in those evaluated in Appendix B tend to infinity for the particular cases $\tau_0\beta_n^2 = 1$, $\tau_1\beta_n^2 = 1$, $\tau_0\alpha_n^2 = 1$, or $\tau_1 = \tau_0$. Such singularities are meaningless physically. A proper solution can be found for each case and no discontinuity occurs.

As it was mentioned, the Lorentz body force has the non-potential part in the considered case. Taking it theoretically, the complete equation of motion (A7) should be solved in order to evaluate the pressure distribution. However, the aim of this paper is to assess the pressure variations and to this end the rigorous solution seems to be unnecessary. One should point out the following.

(1) The liquid is initially at rest. The magnetic pressure is balanced by the static one. The only motion is due to the non-potential part of the Lorentz force.

(2) Only the potential part of $D\mathbf{u}/Dt$ can alter the static pressure.

(3) The kinetic energy is small compared with the magnetic pressure, at least as far as the restriction $Re_m \ll 1$ holds.

Taking this into account the pressure variations are to be assessed from the magnetic Bernoulli equation (2).

EFFETS THERMIQUES ET DE PRESSION DANS UN METAL LIQUIDE DUS AU COLLAPSUS D'UN CHAMP MAGNETIQUE EXTERNE

Résumé—Les équations de l'approximation MHD montrent qu'un champ magnétique non uniforme induit un gradient de pression et un courant dans le liquide conducteur. La différence de pression et le changement de température dus au callapsus d'un champ magnétique externe sont dégagés. Des valeurs élevées de ces effets conduisent à conclure que les effets en question peuvent être importants dans les futures centrales nucléaires à fusion dans lesquelles le réfrigérant serait un métal liquide.

DRUCK- UND WÄRME-EFFEKTE IN FLÜSSIGEM METALL INFOLGE DES ZUSAMMENBRUCHS EINES EXTERNEN MAGNETISCHEN FELDES

Zusammenfassung—Die Näherungsgleichungen für MHD zeigen, daß ein ungleichmäßiges Magnetfeld einen Druckgradienten und einen Strom in einer leitfähigen Flüssigkeit induziert. Die Druckdifferenz und die Temperaturänderung infolge des Zusammenbruchs eines externen Magnetfeldes werden abgeschätzt. Dabei ergeben sich beachtliche Werte, was zu der Schlußfolgerung führt, daß die Effekte möglicherweise für zukünftige Fusions-Kraftwerke wichtig werden können, wo flüssiges Metall ein mögliches Kühlmittel ist.

ВЛИЯНИЕ ОТКЛЮЧЕНИЯ ВНЕШНЕГО МАГНИТНОГО ПОЛЯ НА ИЗМЕНЕНИЕ ДАВЛЕНИЯ И ТЕМПЕРАТУРЫ В ЖИДКОМ МЕТАЛЛЕ

Аннотация—Уравнения в МГД приближении показывают, что неоднородное магнитное поле создает градиент давления и ток в электропроводной жидкости. Оценивается разность давлений и изменение температуры при отключении внешнего магитного поля. Значительная величина этих эффектов позволяет заключить, что они могут иметь существенное значение в будущих энергетических термоядерных станциях с жидкометаллическим теплоносителем.